national accelerator laboratory

TM-436
0402

MEASUREMENT OF THE BETATRON WAVE NUMBER ν BY CLOSED-ORBIT DISTORTION

J.A. MacLachlan and J. Sauer

July 31, 1973

The distortion of the equilibrium orbit of a circular accelerator produced by a short region of perturbed guide field $\Delta(BL)$ is, in linear approximation,

$$x_i = \frac{\Delta(BL)}{B\rho} \frac{\sqrt{\beta_i \ \beta_0}}{2\sin\pi\nu} \cos(|\mu_i - \mu_0| - \pi\nu) \tag{1}$$

where $B\rho$ is the magnetic rigidity, β_i and β_0 are the betatron amplitude functions at the i^{th} sensor and at the perturbed region, respectively; ν is the betatron wave number per turn, and μ_i and μ_0 are the betatron phase at the i^{th} sensor and the perturbation respectively. Therefore, one can measure the ν value by observing the closed orbit distortion produced by a perturbation at a known location. The magnitude of the perturbation need not be known, since ν is available from the phase advance alone.

The determination of ν from the orbit has some advantages over techniques based on observation of free betatron oscillations and is, in some ways, a complementary approach. For a perfect linear machine both methods measure precisely the same quantity; the choice of technique depends on the relative ease and precision of the measurements. In this case, the principal advantage of the static orbit distortion is that it provides the integer part of the tune. The free oscillation method, on the other hand, can get extremely accurate measurements of the fractional part of ν by observing a large number of turns (limited only by the coherence time implied by $\Delta p/p$). In a real accelerator, however, the quantity measured and the measurability are diffently affected in the two methods by non-linear components of the magnetic field. For example, free oscillations will lock onto third or quarter integral values according to the strength of corresponding harmonics of the sextupole and octupole components, respectively. Coherence times are limited by amplitude and momentum dependence of ν arising from average octupole and sextupole fields, respectively. Furthermore, in highly non-linear fields, an oscillation of easily observed amplitude may not be stable. From such effects useful information about the non-linearities may be inferred, but the straight-forward determination of the linear tune is frustrated.

The static orbit distortion method, however, gets around most of the problems. Strong, non-linear fields cause difficulty by making a complicated closed orbit which does not satisfy equation (1). To the extent that the non-linear fields are random, their effect is "noise", which is removed in the fitting of the data to the form of (1). Azimuthal harmonics near ν will change the phase advance per turn and hence the measured ν. Largely compensating for the fact that the phase advance is measured over only a single turn is the fact that the orbit is measured with many beam position detectors. Therefore, the measurement can be rather accurate in principle.

Application to the Main Ring

In the Main Ring, both the correction dipoles and the beam position sensors for each plane are located immediately after the quadrupoles focusing in that plane. One has, therefore, in equation (1) the simplification $\beta_0 \equiv \beta_i$ for all i. By measuring the phase advance from the correction dipole used to provide the orbit distortion ($\mu_0 = 0$) equation (1) may be written

$$x_i = \frac{\Delta(B\ell)}{B\rho} \frac{\beta_0}{2\sin\pi\nu} \cos\nu(\theta_i - \pi) \quad (0 \le \theta_i \le 2\pi) \qquad (2)$$

where $\theta_i = \mu_i/\nu$. Thus, by fitting the difference between perturbed and unperturbed closed orbit at the 96 normal cell position detectors to a two-parameter function

$$x_i = a\cos\nu(\theta_i - \pi) \qquad (3)$$

one obtains the ν value directly. The non-linear least squares fitting is easily handled by the variable metric minimization method.[1] This method gives an error estimate as a by-product so that the reliability of the ν value obtained can be inferred without multiple trials or exhaustive examination of the data. The VMM[2] code has been implemented on the Main-Ring control computer.

Test Cases

The main-ring closed orbit correction program[3] was used to generate an ideal deformed orbit at a known tune of ν = 20.27894077 and the same orbit with an addition of 20% rms random noise in the x_i. Also, a set of measured main-ring data with a 4mm amplitude was analyzed:

Data	fit ν	actual ν	Absolute Error
ideal	20.2787$\pm$.013	20.2789	.0002
20% rms noise	20.2908$\pm$.024	20.2789	.0119
4mm Main Ring	20.2702$\pm$.043	--	--

If the first two entries are indicative, the error figure is conservative. The quality of the real data was estimated by removing the effect of the perturbation using the closed orbit program and the measured ν value. The rms amplitude of the residual distortion was .8mm (20% of the original) with very little 20th harmonic content.

A test of a preliminary version of the program in the Main-Ring control computer has produced very promising results. The beam was kinked vertically over a range from 50% extinction in one sense, to 50% extinction in the opposite sense. The tune measured was $\nu_v = 20.22$ for the entire range. Further trials with more decimal places in the result and numerical error estimate are planned. At the Brookhaven AGS this technique has been used more or less routinely with a reported[4] absolute accuracy of .01.

The ν measurement is most straight-forward for the vertical plane during the injection front porch. For measurements during acceleration, the perturbation must be ramped because the guide field changes substantially during the 40 m sec data collection time of the position detector system. For high fields one of the extraction bumpsmagnets would have to be used to produce an adequate orbit kink. In the horizontal plane, one must either disable radial feedback or else remove the x_p component from the measured orbit distortion and make the corresponding chromaticity correction to ν. None of these difficulties constitute more than minor obstacles to the general application of the method. With suitable software in the main-ring system it would be possible to make tune measurements anytime in the accelerator cycle. The time required for a measurement would be only a few seconds and the resulting accuracy would be about .01 in ν. Such a capability should be an asset in routine tuning, especially during extraction, as well as in diagnostics.

References

1. W.C. Davidon, ANL Report, ANL 5590 (Rev 2)

2. ANL System/360, Library Routine, ANL Z0132 as modified by Keith Rich

3. J.A. MacLachlan, IEEE Trans. on Nucl. Sci., NS-20, No. 3, 892 (1973)

4. M.Q. Barton, R. Frankel, and M. Month, Rev. Sci. Instrum. 40, 1386 (1969)